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Curvatures and potential of M–theory in $D = 4$ with fluxes and twist

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Abstract: We give the curvatures of the free differential algebra (FDA) of M-theory compactified to $D = 4$ on a twisted seven–torus with the 4-form flux switched on. Two formulations are given, depending on whether the 1-form field strengths of the scalar fields (originating from the 3-form gauge field $\hat{A}^{(3)}$) are included or not in the FDA. We also give the bosonic equations of motion and discuss at length the scalar potential which emerges in this type of compactifications. For flat groups we show the equivalence of this potential with a dual formulation of the theory which has the full $E_{7(7)}$ symmetry.

Keywords: M-Theory, Superstring Vacua, Supergravity Models.

Contents

1. Introduction

Higher dimensional theories of supergravity include $(p+1)$-form gauge fields which may couple to p-branes, as well as $(D-3-p)$-form magnetic potentials which have "magnetic" $(D-4-p)$-branes as sources (for a review see [1]). When these theories are compactified to lower-dimensions, S and T-dualities, as well as extended supersymmetry, give rise to non–compact global symmetries [3] called U–dualities [4]. Of particular interest is the case in which the manifold of compactification is a "generalized" Calabi-Yau manifold (in which case the $\mathrm{SU}(N)$ holonomy is replaced by an $\mathrm{SU}(N)$ structure); a variety of fluxes are introduced and these result in "massive" deformations of the original theory without fluxes [5]–[10].

In the low-energy effective supergravity theory these massive deformations can, in most cases, be interpreted as gauged supergravities, in which the non-trivial gauge group (i.e. the gauge symmetry under which some of the p-forms are "charged") is a remnant of the "continuous" U-duality symmetry of the effective supergravity theory (for a review see [11, 12]).

However, although this is the correct description when the bosonic sector of the theory contains only spin 1 and 0 fields (other than the graviton), the gauge structure exhibit a more general form than that of an ordinary Lie algebra, in the case in which the gauge potentials of rank higher than one occur in the theory. This generalized gauge structure is called free differential algebra (FDA) [13 – 15].

2. M-theory FDA

The prototype of this phenomenon is the gauge structure of eleven-dimensional supergravity [2] compactified on a twisted seven-torus with a 4-form flux turned on [16]–[34]. This is a "massive" deformation of the original $D = 11$ supergravity theory compactified on T^7, considered by Cremmer and Julia in their derivation of the $N = 8$, $D = 4$ supergravity. However the deformation occurs before seven antisymmetric tensor fields $B_{\mu\nu I}$ have been dualized into the scalars $\tilde{B}^I$ and so the gauge structure is not encoded in a Lie algebra which is a subalgebra of $\mathrm{E}_{7(7)}$.

The resolution of this puzzle was found in [32, 34] by noting that the 28 gauge potentials $A^I_\mu = g^I_\mu$, $A_{\mu IJ}$ and the seven antisymmetric tensors $B_{\mu\nu I}$ have the structure of a free differential algebra where the curvature 2-forms of the (would be) Lie algebra part are:[1]

$$F^I = dA^I + \frac{1}{2}\tau_{JK}{}^I A^J \wedge A^K \,, \tag{2.1}$$

$$F^{(0)}_{IJKL} = -g_{IJKL} \,, \tag{2.2}$$

$$\mathscr{F}_{IJ} = \mathscr{D}^{(\tau)} A_{IJ} - 6\, g_{IJKL}\, A^K \wedge A^L - \frac{1}{2}\tau_{IJ}{}^K B_K \,, \tag{2.3}$$

where the connection of the $\mathscr{D}^{(\tau)}$ covariant derivative is:

$$\omega^I{}_J = -\tau_{JK}{}^I A^K \,; \qquad \omega_I{}^J = \tau_{IK}{}^J A^K \,, \tag{2.4}$$

and g_{IJKL} is the non trivial flux in internal space.

Since B_K enter the expression of $\mathscr{F}_{IJ}$, this is not an ordinary Lie algebra. Moreover the B_I curvature is:

$$H_I = \mathscr{D}^{(\tau)} B_I + 4\, g_{IJKL}\, A^J \wedge A^K \wedge A^L + 2\, F^J \wedge A_{JI} \,, \tag{2.5}$$

which indicates that g_{IJKL} is a non-trivial cocycle of the FDA.

There is a further space-time 3-form $\mathscr{A}^{(3)}_{\mu\nu\rho}$ whose field strength reads:

$$F^{(4)} = d\mathscr{A}^{(3)} - g_{IJKL}\, A^I \wedge A^J \wedge A^K \wedge A^L - B_I \wedge F^I \,. \tag{2.6}$$

Equations (2.1), (2.3), (2.5), (2.6) are the 4-dimensional reduction of the eleven dimensional equation:

$$\hat{F}^{(4)} = d\hat{A}^{(3)} \,, \tag{2.7}$$

when the scalars coming from the 3-form are set to zero. Here the hats refer to the eleven dimensional forms. Equation (2.2) is a consequence of the reduction of the eleven dimensional vanishing torsion.

We anticipate here that the above curvatures have the same structure as the ones in [34] with the exception of the last terms in eqs. (2.5), (2.6), which of course are absent in the zero-curvature case.

[1] In the present work we have changed the normalizations of g_{IJKL} and B_I with respect to [34], so that the following formulae can be obtained from the corresponding ones in [34] through the substitution: $g \to -12\, g$ and $B_I \to -B_I/2$.

The main result here is that eqs. (2.1), (2.2), (2.3), (2.5) and (2.6) give the complete FDA for the non-zero curvature case. The Bianchi identities involving the non-vanishing curvatures, imply the constraints on ${\tau_{IJ}}^K$, g_{IJKL} [28] which were obtained as integrability conditions in the zero-curvature case [32] and which read:

$$\tau_{[IJ}{}^{K}\, \tau_{M]K}{}^{L} = 0\,, \tag{2.8}$$

$$\tau_{[IJ}{}^{K}\, g_{MNL]K} = 0\,. \tag{2.9}$$

The full set of Bianchi identities is:

$$\begin{aligned}
\mathscr{D}\mathscr{F}_{IJ} + \frac{1}{2}\tau_{IJ}{}^{K}\, H_K &= 0\,, && (2.10)\\
\mathscr{D}F^I &= 0\,, && (2.11)\\
\mathscr{D}H_I + 2\, F^J \wedge \mathscr{F}_{JI} &= 0\,, && (2.12)\\
\mathscr{D}^{(\tau)} F^{(0)}_{IJKL} + 6\,\tau_{IJ}{}^{M}\, g_{KLMN}\, A^N &= 0\,, && (2.13)
\end{aligned}$$

where the covariant derivative $\mathscr{D}$ acting on the field strengths is defined as follows:

$$\mathscr{D}\begin{pmatrix} F^I \\ F_{IJ} \end{pmatrix} = d\begin{pmatrix} F^I \\ F_{IJ} \end{pmatrix} - \begin{pmatrix} \omega^I{}_K & 0 \\ \omega_{IJ|K} & \omega_{IJ}{}^{LN} \end{pmatrix}\begin{pmatrix} F^K \\ F_{LN} \end{pmatrix}, \tag{2.14}$$

the new connection being defined as:

$$\begin{aligned}
\omega^I{}_K &= \tau_{KL}{}^{I}\, A^L\,,\\
\omega_{IJ|K} &= -12\, g_{IJKL}\, A^L + 3\,\delta^L_{[I}\,\tau_{J]K}{}^{M}\, A_{LM}\,,\\
\omega_{IJ}{}^{LN} &= -2\,\delta^{[L}_{[I}\,\tau_{J]K}{}^{M]}\, A^K\,.
\end{aligned} \tag{2.15}$$

The Bianchi identity for $F^{(4)}$ is trivial since we are in $D = 4$.

Integrability of eqs. (2.10), (2.11) and (2.12) can be checked by further applying to them the $\mathscr{D}$ operator. As explained in reference [32], the set of curvatures (F^I, $\mathscr{F}_{IJ}$) do not form an ordinary Lie algebra since the "structure constants" ${f_{\Lambda\Sigma}}^\Gamma$ entering the quadratic part of:

$$\mathscr{F}^\Lambda = dA^\Lambda + \frac{1}{2}\, f_{\Gamma\Sigma}{}^{\Lambda}\, A^\Gamma \wedge A^\Sigma + m^{\Lambda I}\, B_I\,, \tag{2.16}$$

do not satisfy the Jacobi identities of a Lie algebra:

$$f_{[\Lambda\Sigma}{}^{\Gamma}\, f_{\Delta]\Gamma}{}^{\Pi} \neq 0\,, \tag{2.17}$$

because of the trilinear term in the definition (2.5) of H_I.

Even when $g_{IJKL} = 0$ the 28-dimensional Lie algebra gauged by the vectors A^I, A_{IJ} is not a subalgebra of $\mathrm{E}_{7(7)}$, as explained in [34]. This is a consequence of the phenomenon of "dualization of dualities" discussed in [35].

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3. FDA, including scalar fields

To make contact with the original work of ref. [16] and their definition of curvatures (called G in [16]), it is useful to include the scalar fields:

$$C_{IJK} = A_{IJK}\,, \tag{3.1}$$

and also an "internal" curvature 0-form :

$$F^{(0)}_{IJKL} = -g_{IJKL} - \frac{3}{2}\,\tau_{[IJ}{}^{M}\,C_{KL]M}\,. \tag{3.2}$$

Then we may include a curvature 1-form:

$$F^{(1)}_{IJK} = \mathscr{D}^{(\tau)}C_{IJK} - 4\,g_{IJKL}\,A^{L} - \tau_{[IJ}{}^{L}A_{K]L}\,, \tag{3.3}$$

and a modified curvature 2-form:

$$F^{(2)}_{IJ} = \mathscr{F}_{IJ} - 3\,C_{IJK}\,F^{K}\,, \tag{3.4}$$

and no other modifications in $H^{(3)}_{I}$ and $F^{(4)}$. The new Bianchi identities are:

$$\begin{aligned}
\mathscr{D}^{(\tau)}F^{(0)}_{IJKL} &= -\frac{3}{2}\tau_{[IJ}{}^{N}\,F^{(1)}_{KL]N}\,, &(3.5)\\
\mathscr{D}^{(\tau)}F^{(1)}_{IJK} + \tau_{[IJ}{}^{N}\,F^{(2)}_{K]N} - 4\,F^{L}\wedge F^{(0)}_{LIJK} &= 0\,, &(3.6)\\
\mathscr{D}^{(\tau)}F^{(2)}_{IJ} + \frac{1}{2}\,\tau_{IJ}{}^{L}\,H_{L} + 3\,F^{(1)}_{IJK}\wedge F^{K} &= 0 &(3.7)\\
\mathscr{D}^{(\tau)}H_{I} + 2\,F^{J}\wedge F^{(2)}_{JI} &= 0\,, &(3.8)
\end{aligned}$$

(note that $F^{I}\wedge F^{(2)}_{IJ} = F^{I}\wedge\mathscr{F}_{IJ}$ since $F^{I}\wedge F^{J}\,C_{IJK} = 0$). These are the Bianchi identities of the curvatures $G^{(0)}_{IJKL}$, $G^{(1)}_{\mu JKL}$, $G^{(2)}_{\mu\nu KL}$, , $G^{(3)}_{\mu\nu\rho L}$ introduced in ref. [16], with

$$G^{(0)} = F^{(0)}\,; \qquad G^{(1)} = F^{(1)}\,; \qquad G^{(2)} = F^{(2)}\,; \qquad G^{(3)} = H\,. \tag{3.9}$$

4. The equations of motion and the potential

The bosonic equations of motion of M-theory can be obtained by varying the lagrangian with respect to the vielbein 1-form V^{a} and the 3-form $\hat{A}$.

The $g_{\mu\nu}$, G_{IJ} and A^{I} field equations come from the eleven dimensional Einstein equations:

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R &= T_{\mu\nu}\,,\\
R_{\mu I} - \frac{1}{2}\,g_{\mu I}\,R &= T_{\mu I}\,,\\
R_{IJ} - \frac{1}{2}\,g_{IJ}\,R &= T_{IJ}\,,
\end{aligned} \tag{4.1}$$

where $g_{\mu I} = G_{IJ}\,A^{J}_{\mu}$ and G_{IJ} are the coordinates of GL(7)/SO(7). The tensor T is the energy momentum tensor of the 4-form. Incidentally we remark that in this formulation the R-symmetry of the corresponding $N = 8$ supergravity is Spin(7), the eleven dimensional gravitino gives rise to eight gravitinos which are in the eight-dimensional spinorial representation and to spin 1/2 which transform in the **8** + **48** of the same group.

The 3-form field equations read as follows:[2]

$$d \star F^{(4)} = \frac{1}{4} F^{(4)} \wedge F^{(4)} . \tag{4.2}$$

Since in this paper we are mainly concerned with the general form of the scalar potential coming from the twist and the fluxes, we will carefully analyze this equation only for those entries which receive contributions from the scalar potential. Let us write the dual of the field equations originating from the Euler-Lagrange equations for $\mathcal{A}_{\mu\nu\rho}$ and C_{IJK}. The first equation allows us to integrate out the $\mathcal{A}_{\mu\nu\rho}$ field in a manner which we shall explain in a moment. This integration gives an extra contribution to the scalar potential coming from the Chern-Simons term. The second equation contains the derivative of the vacuum energy with respect to C_{IJK} and contributes to the equation of motion of the C_{IJK} scalar.

Let us define the following 4-D scalar quantity:

$$P = \frac{1}{\sqrt{-g}} \epsilon^{\mu_1 \dots \mu_4} F^{(4)}_{\mu_1 \dots \mu_4} , \tag{4.3}$$

where $F^{(4)}_{\mu_1 \dots \mu_4}$ was defined in (2.6). For the purpose of computing the scalar potential, only the $d\mathscr{A}^{(3)}$ part of $F^{(4)}$ will be relevant. The $\mathcal{A}_{\mu\nu\rho}$ field equation then reads:

$$d(V_7 \, P) = -\frac{1}{4} F^{(1)}_{IJK} F^{(0)}_{PQRS} \epsilon^{IJKPQRS} . \tag{4.4}$$

For the purpose of computing the potential, only the $\mathscr{D}^{(\tau)} C_{IJK}$ term in $F^{(1)}_{IJK}$ is relevant. Equation (4.4) implies that its right hand side is a closed form. In fact the crucial ingredient is that the term $F^{(1)}_{IJK} F^{(0)}_{PQRS} \epsilon^{IJKPQRS}$ is an exact form on the twisted torus with fluxes, and it can be written as

$$F^{(1)}_{IJK} F^{(0)}_{PQRS} \epsilon^{IJKPQRS} = -d \left(C_{IJK} \, (g_{PQRS} + \frac{3}{4} \tau^{N}_{[PQ} \, C_{RS]N}) \epsilon^{IJKPQRS} + \tilde{g} \right) , \tag{4.5}$$

where the integration constant $\tilde{g}$ [36] is actually related to the dual gauge algebra in the $E_{7(7)}$ covariant formulation described in [34]. From this we get the value of $V_7 \, P$ to be:

$$V_7 \, P = \frac{1}{4} \left(C_{IJK} \left(g_{LPQR} + \frac{3}{4} \tau^{N}_{[LP} \, C_{QR]N} \right) \epsilon^{IJKLPQR} + \tilde{g} \right) . \tag{4.6}$$

Note the important identity:

$$\frac{\delta P}{\delta C_{IJK}} = -\frac{1}{4} \epsilon^{IJKLPQR} F^{(0)}_{LPQR} . \tag{4.7}$$

Let us now turn to considering the equation of motion for the C_{IJK} fields. They read:

$$\begin{aligned} \partial_\mu \left(V_7 \sqrt{-g} G^{I_1 J_1} G^{I_2 J_2} G^{I_3 J_3} \, g^{\mu\nu} \, \partial_\nu C_{J_1 J_2 J_3} \right) &= -\frac{3}{2} \frac{1}{7!} \epsilon^{\mu\nu\rho\lambda} F_{IJKP} F_{\mu\nu\rho\lambda} \epsilon^{I_1 I_2 I_3 IJKP} - \\ &\quad - \frac{1}{2} V_7 \sqrt{-g} \, \tau_{PQ}{}^{[I_1} F^{I_2 I_3]PQ} . \end{aligned} \tag{4.8}$$

[2]For the eleven dimensional equations we are using the conventions and notations of reference [13]

By using equations (4.6) and (4.7) and the fact that:

$$\frac{\delta(F^{(0)}_{IJKL}\,F^{(0)\,IJKL})}{\delta C_{PQR}} = -3\,\tau^{[P}_{IJ}\,F^{(0)QR]IJ}\,, \tag{4.9}$$

equation (4.8) can be rewritten in the form:

$$\partial_\mu\left(V_7\sqrt{-g}G^{I_1J_1}G^{I_2J_2}G^{I_3J_3}\,g^{\mu\nu}\,\partial_\nu C_{J_1J_2J_3}\right) = \sqrt{-g}\,\frac{\delta V}{\delta C_{I_1I_2I_3}}\,, \tag{4.10}$$

where the C_{IJK}-dependent part of the potential is:

$$\begin{aligned} V_C = {}& \frac{3}{16}\frac{1}{7!}\frac{1}{V_7}\left(C_{IJK}\left(g_{LPQR}+\frac{3}{4}\tau^N_{[LP}\,C_{QR]N}\right)\epsilon^{IJKLPQR}+\tilde{g}\right)^2 + \\ & +\frac{1}{6}V_7\,F^{(0)}_{IJKL}\,F^{(0)}_{MNPQ}\,G^{IM}\,G^{JN}\,G^{KP}\,G^{LQ}\,, \end{aligned} \tag{4.11}$$

where $F^{(0)}_{IJKL}$ is given in eq. (3.2). One can easily compute the scalar potential in the Einstein frame by noting that

$$g_{\mu\nu} = \frac{1}{V_7^2}\,g^E_{\mu\nu}\,. \tag{4.12}$$

Therefore in this frame, the potential becomes multiplied by an overall $(V_7)^{-2}$.

The full scalar potential in the Einstein frame is thus obtained by adding to V_C the Scherk-Schwarz purely G-dependent part originating from the eleven-dimensional Einstein term. It is useful to write the entire potential as the following sum:

$$V = V_E + V_K + V_{C-S}\,, \tag{4.13}$$

where the three terms on the right hand side originate from the eleven dimensional Einsten, kinetic and Chern-Simons terms respectively, and are found to have the following expression:

$$\begin{aligned} V_E &= \frac{1}{V_7}\left(2\,G^{KL}\,\tau_{KJ}{}^I\,\tau_{LI}{}^J + G_{II'}\,G^{JJ'}\,G^{KK'}\,\tau_{JK}{}^I\,\tau_{J'K'}{}^{I'}\right), \\ V_K &= \frac{3}{16}\frac{1}{7!}\frac{1}{V_7}\left(g_{IJKL}+\frac{3}{2}\tau^R_{[IJ}C_{KL]R}\right)\left(g_{MNPQ}+\frac{3}{2}\tau^R_{[MN}C_{PQ]R}\right)G^{IM}G^{JN}G^{KP}\,G^{LQ}\,, \\ V_{C-S} &= \frac{1}{6}\frac{1}{V_7^3}\left(C_{IJK}\left(g_{LPQR}+\frac{3}{4}\tau^N_{[LP}\,C_{QR]N}\right)\epsilon^{IJKLPQR}+\tilde{g}\right)^2\,. \end{aligned} \tag{4.14}$$

Note that for $\tau = g = 0$ we just get a positive cosmological constant, as noted in [36].

5. Flat group vacua of the potential

The scalar potential in (4.13) and (4.14) has the property that $V_K \geq 0$, $V_{C-S} \geq 0$ while V_E has no definite sign [16]. Therefore in general we may have vacua with different signs of the cosmological constant.

A particular appealing class of models, which correspond to "no-scale" supergravities [37], are obtained for those gaugings for which $V_E \geq 0$. This defines a "flat group" and implies restrictions of the τ matrices. These were described in the pioneering paper of ref. [16] for $g_{IJKL} = \tilde{g} = 0$.

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It is our aim to show here that for any flat group at $g_{IJKL} = \tilde{g} = 0$ there is a new flat gauge algebra with additional structure constants related to g and $\tilde{g}$. We first discuss this situation by looking at the flat vacua of our potential and, in the next section, from the point of view of the gauge structure.

To find extrema with zero cosmological constant of the scalar potential we first analyze the equation $\delta\mathscr{L}/\delta C_{IJK} = 0$. Because of the properties (4.7) and (4.9) this is ensured by setting $F_{IJKL} = 0$. The contribution of the C_{IJK} field to the vacuum energy vanishes if $P = 0$ at $F_{IJKL} = 0$. This can be always obtained by a particular choice of $\tilde{g}$ as a function of g. The necessary conditions for a flat vacuum can be then summarized as follows:

$$F^{(0)}_{IJKL} = 0 \quad \Leftrightarrow \quad g_{IJKL} + \frac{3}{2}\,\tau_{[IJ}{}^{P}\,C_{KL]P} = 0\,, \tag{5.1}$$

$$P = 0 \quad \Leftrightarrow \quad C_{MNR}\left(g_{IJKL} + \frac{3}{4}\,\tau_{[IJ}{}^{P}\,C_{KL]P}\right)\epsilon^{MNRIJKL} + \tilde{g} = 0\,. \tag{5.2}$$

The second equation can also be written as the following condition on $\tilde{g}$:

$$\tilde{g} = \frac{3}{4}\,C^{0}_{MNR}\,\tau_{[IJ}{}^{P}\,C^{0}_{KL]P}\,\epsilon^{MNRIJKL}\,, \tag{5.3}$$

where C^0_{IJK} is a solution of equation (5.1) and thus depends on g_{IJKL}. This equation ensures that the G_{IJ} moduli equations are the same as in the $g = \tilde{g} = 0$ case, because the F-contribution to the energy-momentum tensor vanishes in these vacua.

To make a concrete example, let us consider the case in which $I = 0, i$, $i = 1, \ldots, 6$ with $\tau_{IJ}{}^{K} = \tau_{0i}{}^{j}$, zero otherwise, and $g_{IJKL} = g_{0ijk}$, zero otherwise. In this case $\tau_{0i}{}^{j} = T_j{}^i$ is chosen to be an antisymmetric matrix of rank 3 which can be set in the form:

$$T_i^{\,j} = \begin{pmatrix} m_1\,\epsilon & 0 & 0 \\ 0 & m_2\,\epsilon & 0 \\ 0 & 0 & m_3\,\epsilon \end{pmatrix}; \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{5.4}$$

In this context the equation (5.1) becomes $F^{(0)}_{0ijk} = 0$ which fixes all C_{ijk} fields but not the C_{0ij} scalars. The C_{0ij} fields give masses to the A_{ij} vector fields with the exception of the three entries $(ij) = (1,2), (3,4), (5,6)$. Therefore three of the C_{0ij} scalar remain massless moduli. The G_{IJ}-sector gives, as discussed in reference [16], four additional massless scalars, of which two are the volume V_7 and G_{00} and two other come from internal components of the metric.

If one further discusses the spectrum of the remaining fields, the six vector A_{i0} are eaten by the six antisymmetric tensors B_i because of the magnetic mass term in the FDA (2.1). An additional massless scalar comes from the massless 2-form B_0 and finally an additional massless vector come from the A^0 Kaluza-Klein vector. The other six A^i vectors become massive because of the twisting of the torus. We conclude that in this theory there are always eight massless scalars and four massless vectors, in agreement with [16]. The effect of turning on g and $\tilde{g}$ is not of giving extra masses, but of shifting the v.e.v. of the C_{IJK} fields. This can be understood by an extension of the flat group where g and $\tilde{g}$ play the role of additional structure constants. In the next section we will recover this result as well as the form of the potential, from the underlying duality symmetry of the dual formulation of the theory, in which all antisymmetric tensors B_I are dualized into scalars $\tilde{B}^I$ and the $E_{7(7)}$ symmetry is recovered.

6. The dual gauge algebra and its scalar potential

We now interpret the above result in the usual formulation of the four dimensional theory based on the flat gauging. From the results of [34] this amounts to dualize those vector fields which participate to the anti-Higgs mechanism, in our case they are the A_{0i} 1-forms, which are therefore replaced by their A^{0i} magnetic duals. The dual gauge algebra therefore contains the following 28 generaors:

$$W^{ij},\, W_i,\, Z_i,\, Z_0\,. \tag{6.1}$$

with structure constants obtained from [34, eq. (2.13)]. The first 27 generators form an abelian algebra, and the only non vanishing commutators are those involving Z_0 and given by:

$$\begin{aligned}
[Z_0,\, Z_i] &= T_j{}^j\, Z_j - 12\, g_{0ijk}\, W^{jk} + \tilde{g}\, W_i \\
[Z_0,\, W^{pq}] &= 2\, T_i{}^{[p}\, W^{q]i} - 12\, g_{0ijk}\, \epsilon^{ijkpql}\, W_l \\
[Z_0,\, W_i] &= T_j{}^j\, W_j\,,
\end{aligned} \tag{6.2}$$

where with respect to [34] the redefinition $g \to -12\, g$ was made. This algebra defines a flat subalgebra of $E_{7(7)}$ which fits the class of models duscussed by Cremmer, Scherk and Schwarz in [17] and in [18], as it was shown in [29] and in [34]. The gauged supergravity interpretation was given in [23] and the corresponding gauge algebra is the semidirect product of a U(1) by a 27-dimensional abelian algebra and is contained in the branching of $E_{7(7)}$ with respect to $E_{6(6)} \times O(1,1)$:

$$\mathbf{133} \to \mathbf{1}_0 + \mathbf{78}_0 + \mathbf{27'}_{+2} + \mathbf{27}_{-2}\,. \tag{6.3}$$

To compare with the geometrical twist we further branch $E_{6(6)}$ with respect to $SL(6) \times SL(2)$:

$$\mathbf{78} \to (\mathbf{35},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{20},\mathbf{2})\,, \tag{6.4}$$

$$\mathbf{27} \to (\mathbf{15'},\mathbf{1}) + (\mathbf{6},\mathbf{2})\,. \tag{6.5}$$

Our gauging corresponds to the following choice of the "twist matrix" (see [29] and equation (2.9) of [34]):

$$Z_0 = -\frac{2}{3} T_i{}^j\, t_j{}^i + g_{0ijk}\, t^{ijk} + \frac{1}{9}\tilde{g}\, t_0\,, \tag{6.6}$$

where we have used the notations introduced in [34]. Here $t_i{}^j$ are the generators of the maximal compact subgroup of SL(6), namely SO(6), while t^{ijk} and t_0 are nilpotent generators: the former belong to the $(\mathbf{20},\mathbf{2})$ representation in (6.4) with positive grading with

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respect to the $\mathfrak{o}(1,1)$ generator of SL(2) and the latter is the nilpotent generator of SL(2) with positive grading with respect to the same generator. In the same framework we now discuss the form of the scalar potential, which is expected not to depend on the dualization procedure. In the dual formulation this potential is given by [18, 23, 24]:

$$V = e^{-6\,\phi}\left(\frac{1}{2}\,(P_{0\,\hat{i}}{}^{\hat{j}}) + \frac{1}{6}\,(P_{0\,\hat{i}\hat{j}\hat{k}})^2 + (P_0{}^0)^2\right) = V_E + V_K + V_{C-S}\,, \tag{6.7}$$

where ϕ is the modulus associated with the 0^{th} internal dimension of compactification,the hatted indices are rigid SO(6) indices, while the quantity P_0 has value in the 42-dimensional non-compact part of the $\mathfrak{e}_{6(6)}$ Lie algebra and represents the vielbein of the five-dimensional σ-model $\mathrm{E}_{6(6)}\,/\,\mathrm{USp}(8)$. It is defined as follows:

$$P_0 = (L^{-1}\,Z_0\,L)_{|\text{non-compact}}\,, \tag{6.8}$$

where L is the five-dimensional coset representative which, using the solvable Lie algebra parametrization of $\mathrm{E}_{6(6)}\,/\,\mathrm{USp}(8)$, can be directly written in terms of our scalar fields as follows:

$$L = e^{\tilde{B}^0\,t_0}\,e^{\frac{1}{6}\,C_{ijk}\,t^{ijk}}\,\mathbb{E}\,;\qquad \mathbb{E}\in\frac{\mathrm{GL}(6)}{\mathrm{SO}(6)}\,. \tag{6.9}$$

Direct computation shows that:

$$\begin{aligned}
P_{0\,\hat{i}\hat{j}} &= T_i{}^j\,\mathbb{E}^{-1\,i}_{(\hat{i}}\,\mathbb{E}_{j|\hat{j})}\,,\\
P_{0\,\hat{i}\hat{j}\hat{k}} &\propto \left(g_{0ijk} + \frac{3}{4}\,T_{[i}{}^n\,C_{jk]n}\right)\,\mathbb{E}^{-1\,i}_{\hat{i}}\,\mathbb{E}^{-1\,j}_{\hat{j}}\,\mathbb{E}^{-1\,k}_{\hat{k}}\,,\\
P_0{}^0 &\propto \epsilon^{lmnijk}\,C_{lmn}\left(g_{0ijk} + \frac{3}{8}\,T_{[i}{}^n\,C_{jk]n}\right) + \tilde{g}\,.
\end{aligned} \tag{6.10}$$

In this language the eight massless modes come from $\tilde{B}^0$, three from C_{0ij}, one from ϕ and three from the metric G_{ij}. The latter can be understood from the fact that under SO(6) these moduli transform in the $\mathbf{1}+\mathbf{20}'$ and the $\mathbf{20}'$ has two vanishing weights.

7. Conclusions and outlook

In the present investigation we have presented the full set of (bosonic) curvatures and their Bianchi identities for the free differential algebra underlying M-theory compactified on a twisted seven-torus with the 4-form flux turned on. The resulting curvatures show that the B_I 2-forms receive a mass from the 1-form fields A_{IJ} through the twist matrix τ, see equation (2.3). Moreover the B_I curvature H_I shows that a Green-Schwarz coupling [38] is present which modifies the corresponding Bianchi identity (2.12). In deriving the results of sections 2–4, we have used an expansion of the eleven dimensional 3-form and of its 4-form curvature in terms of the internal twisted torus with fluxes, along similar lines as those discussed in references [19, 28]. By projecting the FDA and the equations of motion on a suitable basis of 1-forms one recovers the main formulae of sections 2–4.

Combined gauge invariance of tensors of different rank has been also considered in a different framework in [39]. We have derived the scalar potential (eqs. (4.13), (4.14)) and discussed the flat group vacua giving rise to a no-scale form of $N = 8$ supergravity [23].

Moreover we have compared the results with a dual formulation of the theory where those vectors participating to the anti-Higgs phenomenon have been dualized together with the seven antisymmetric tensors so that the $\mathrm{E}_{7(7)}$ symmetry is restored. In the dual formulation the scalar potential is given in the form obtained by dimensional reduction from five dimensions [18, 23] in the presence of a suitable $\mathrm{E}_{6(6)}$-twist. It is proven that the two potentials do coincide, giving evidence that the theories are "dual" even in the presence of gaugings. This is however possible only for a restricted gauge algebra where the U(1) compact generator Z_0 is taken in the (rank three) maximal compact subgroup Spin(6) = SU(4) of SL(4), rather than in the rank-4 maximal compact subgroup USp(8) of $\mathrm{E}_{6(6)}$. The missing compact generator lies inside the $\mathfrak{sl}(2)$ subalgebra of $\mathfrak{e}_{6(6)}$ commuting with $\mathfrak{sl}(6)$. It would be interesting to compute the full lagrangian reduced to $D = 4$ and its dual theory with the underlying gauge algebra of reference [34]. This would involve dualization of massive tensors along the same lines of references [40]–[44]

We have not discussed vacua with $F^{(0)}_{IJKL}$ and/or P not vanishing. First of all we may notice that , on general grounds, if we assume $G_{IJ} = \lambda^2\, \delta_{IJ}$, the three terms in (4.14) scale differently; for large λ they behave as $\lambda^{-9}, \lambda^{-15}, \lambda^{-21}$, respectively, so the sign of V_E dominate the potential for large λ.

The Einstein equations, (4.1), may have solutions where the gravitational part is compensated by the energy momentum tensor of the C_{IJK} scalars. The equation of motion, given in (4.8), may have a solution for $F^{(0)}_{IJKL} \neq 0$. We note that these equations are homogeneous in $F^{(0)}_{IJKL}$. The solutions with non flat vacua may be important for cosmological applications.

We also note that in the FDA formulation of the theory the R-symmetry is Spin(7) while in the dual formulation is Spin(6) = SU(4). The eight gravitinos belong to the **8** of Spin(7) in one formulation and to the $\mathbf{4} + \overline{\mathbf{4}}$ of Spin(6) in the dual formulation. When the fermionic sector is included in the theory, the bosonic FDA extends to a "super" FDA, that we call SFDA. The curvatures and Bianchi identities of the full theory will then include the spin-3/2 curvatures and possibly the spin-1/2 curvatures. Their construction is under investigation and will be given elsewhere.

Finally it is possible to compute several truncations of this $N = 8$ theory in order to obtain lower ($N < 8$) local supersymmetry.

The scalar potential of the reduced theory will be obtained in this case by a particular truncation of the original potential [24].

It should be emphasized that there is a subtlety in the computation of the M-theory potential, which is the term V_{C-S} of equation (4.14). This lies in the derivation of the equation of motion of the auxiliary field P in (4.3). In fact the contribution from the Chern-Simons term does not come by merely treating P as an algebraic Lagrange multiplier, but rather by varying the potential $\mathcal{A}_{\mu\nu\rho}$ and solving a differential equation for P, (4.4). Its integration yields equation (4.6), which is in agreement with the result of ref. [45], where

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a different derivation was used. It also gives an expression which vanishes at $F^{(0)}_{IJKL} = 0$ by suitably adjusting the $\tilde{g}$ coupling, equation (5.3). This result is also in agreement with the dual formulation (equation (6.10)), where $\tilde{g}$ plays the role of a structure constant of the gauge Lie algebra given in (6.2).

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